

# Exchange effects in Coulomb quantum plasmas: Dispersion of waves in 2D and 3D mediums

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We describe quantum hydrodynamic equations with the Coulomb exchange interaction for three and two dimensional plasmas. Explicit form of the force densities are derived. We present non-linear Schrodinger equations (NLSEs) for the Coulomb quantum plasmas with the exchange interaction. We show contribution of the exchange interaction in the dispersion of the Langmuir, and ion-acoustic waves. We consider influence of the spin polarization ratio on strength of the Coulomb exchange interaction. This is important since exchange interaction between particles with same spin direction and particles with opposite spin directions are different. At small particle concentrations  $n_0 \ll 10^{25} \text{ cm}^{-3}$  and small polarization the exchange interaction gives small decrease of the Fermi pressure. With increasing of polarization role of the exchange interaction becomes more important, so that it can overcome the Fermi pressure. The exchange interaction also decreases contribution of the Langmuir frequency. Ion-acoustic waves do not exist in limit of large polarization since the exchange interaction changes the sign of pressure. At large particle concentrations  $n_0 \gg 10^{25} \text{ cm}^{-3}$  the Fermi pressure prevails over the exchange interaction for all polarizations. Similar picture we obtain for two dimensional quantum plasmas.

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## I. INTRODUCTION

Exchange interaction is a remarkable example of quantum physical effects. The exchange interaction is related to overlapping of the wave functions of interacting particles. Hence it reveals itself as a short range interaction, even when we consider the exchange part of a long range interaction.

Let us now give a brief historical overview of quantum plasma research tracing contribution of the exchange interaction. Hydrodynamic description of quantum plasmas was considered in 1999 by Kuz'menkov and Maksimov [1] and Kuzelev and Rukhadze [2], where were considered spinless Coulomb quantum plasmas. The self-consistent field approach was in the center of attention of these papers. Nevertheless a general form of the exchange interaction for bosons and fermions were derived in Ref. [1]. In 2000-2001 attention shifted towards spin-1/2 quantum plasmas [3]-[6]. The explicit form of the exchange interaction for the Coulomb and the spin-spin interactions were derived in Ref. [6]. Contribution of these interactions in properties of many-electron atoms was described there.

During period 2001-2007 most of researches considered spinless quantum plasmas [7], [8] (for review see Refs. [9], [10]). A great interest to quantum plasmas of spinning particles arose since 2007 Marklund and Brodin drew attention to the field by their papers [11], [12], some examples we are presented below.

Dispersion properties of spin-1/2 quantum plasmas have been under consideration for many years (see Refs. [4], [11], [13]-[32]). It was shown that magnetic moment (spin) evolution in quantum plasmas leads to new branches of wave dispersion [14], [15], [16], [21], [22], [26], [29]. The last of these Refs. [29] shows dispersion of spin wave in the spin-1/2 two dimensional electron gas. Contribution of the exchange interaction in dispersion properties of spin-1/2 quantum plasmas was considered in 2008 (see Ref. [19]). It was demonstrated that the exchange interactions give potential force field depending on the particle concentration and the spin density. In linear approximation the exchange interactions can be combined with the Fermi pressure, so we have an effective shifted Fermi velocity [19]. Description of spin-plasma wave propagating perpendicular to an external magnetic field was given in 2006 by Vagin et al. in 2006 (see Ref. [14]), Andreev and Kuz'menkov in 2007 (see Ref. [15]), and Brodin et al. in 2008 (see Ref. [16]). In Refs. [14] and [16] the wave is considered in terms of kinetic equation, the quantum hydrodynamics was applied for consideration of the wave in Ref. [15]. Spin propagating parallel to an external magnetic field was obtained by Misra et al. in 2010 (see Ref. [21]) and Andreev and Kuz'menkov in 2011 (see Ref. [26]). Influence of the spin-orbit interaction on these waves was considered in 2011-2012 (see Refs. [22] and [26]).

Charge of particles creating the electric field is essential for the plasma wave existence. However the spin waves can exist in systems of neutral particles, since the magnetic field plays main role in spin wave propagation. It was suggested in Ref. [15] that the spin-1/2 quantum plasmas can support spin waves propagating by magnetic field with no electric field contributing in their propaga-

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tion. Dispersion of such waves was considered in Refs. [15] and [22]. Three branches of these waves were obtained. Dispersion of one of them depends on the Fermi velocity. Hence the exchange interaction gives contribution in this wave dispersion via the shifted Fermi velocity [19]. Spin waves in systems of neutral particles existing due to the long range spin-spin interaction are considered in Ref. [33] in terms of quantum kinetics. This quantum kinetics was derived in Ref. [33] as direct generalization of the method of many-particle quantum hydrodynamics [1], [5], [6], [22], [28], [34], [35]. This kinetics differs from recent generalization of the Wigner kinetics for spinning particles [36]-[41].

All mentioned effects are based on scalar g-factor theories. Effect of tensor g-factor on the spectrum of eigen modes in spin-1/2 quantum plasmas was considered by Vagin et al (see Ref. [20]).

Let us mention excellent applications of exchange interaction in terms of many-particle quantum hydrodynamics. They were presented in Ref. [42] for system of neutral quantum particles with the short range interaction. The famous Gross-Pitaevskii equation, for the inhomogeneous non-ideal Bose-Einstein condensates, together with its analog for ultracold fermions, and their generalizations were derived there.

Recent achievements in field of quantum plasmas with the exchange interactions can be found in Refs. [43] and [44], where authors applied the Wigner kinetics [45].

In this paper we consider contribution of the Coulomb exchange interaction in spectrum of quantum plasma waves including ion-acoustic waves. Recently a model for classic and quantum plasmas including the finite size of ions was developed (see Ref. [46]). Contribution of the finite size of ions in dispersion of ion-acoustic waves was obtained in Ref. [46].

This paper is organized as follows. Two fluid quantum hydrodynamics for Coulomb plasmas with the exchange interaction is presented in Sec. II. In Sec. III we give applications of the developed model to the Langmuir and ion-acoustic waves in two- and three dimensional quantum plasmas. In Sec. IV brief summary of obtained results is presented.

## II. MODEL

In this paper we present a set of quantum hydrodynamic equations for spinless Coulomb quantum plasmas without derivation. Method of direct derivation of the quantum hydrodynamic equations from many-particle Schrodinger equation can be found in Refs. [1] (Coulomb interaction with the exchange part for Bose and Fermi particles), [5], [6], [34] (spin-1/2 charged particles), [35] (charged particles bearing electric dipole moment), [47] (spinless semi-relativistic quantum plasmas). Some details of derivation can be also found in Ref. [42] dedicated to quantum neutral particles with exchange interactions.

The QHD equations for electron subsystem in electron-

ion quantum plasmas

$$\partial_t n_e + \nabla(n_e \mathbf{v}_e) = 0, \quad (1)$$

and

$$\begin{aligned} m_e n_e (\partial_t + \mathbf{v}_e \nabla) \mathbf{v}_e + \nabla p_e - \frac{\hbar^2}{2m_e} n_e \nabla \left( \frac{\Delta n_e}{n_e} - \frac{(\nabla n_e)^2}{2n_e^2} \right) \\ = q_e n_e \left( \mathbf{E}_{ext} + \frac{1}{c} [\mathbf{v}_e, \mathbf{B}_{ext}] \right) - q_e^2 n_e \nabla \int G(\mathbf{r}, \mathbf{r}') n_e(\mathbf{r}', t) d\mathbf{r}' \\ - q_e q_i n_e \nabla \int G(\mathbf{r}, \mathbf{r}') n_i(\mathbf{r}', t) d\mathbf{r}' + \mathbf{F}_{C,e}, \end{aligned} \quad (2)$$

and the QHD equations for ions

$$\partial_t n_i + \nabla(n_i \mathbf{v}_i) = 0, \quad (3)$$

and

$$\begin{aligned} m_i n_i (\partial_t + \mathbf{v}_i \nabla) \mathbf{v}_i + \nabla p_i - \frac{\hbar^2}{2m_i} n_i \nabla \left( \frac{\Delta n_i}{n_i} - \frac{(\nabla n_i)^2}{2n_i^2} \right) \\ = q_i n_i \left( \mathbf{E}_{ext} + \frac{1}{c} [\mathbf{v}_i, \mathbf{B}_{ext}] \right) - q_i^2 n_i \nabla \int G(\mathbf{r}, \mathbf{r}') n_i(\mathbf{r}', t) d\mathbf{r}' \\ - q_e q_i n_i \nabla \int G(\mathbf{r}, \mathbf{r}') n_e(\mathbf{r}', t) d\mathbf{r}' + \mathbf{F}_{C,i}. \end{aligned} \quad (4)$$

Set of equations (1), (2) and (3), (4) are coupled to each other by means of the last terms in the Euler equations (2) and (4). In equation set (1)-(4) we assumed that thermal pressure is isotropic:  $p_a^{\alpha\beta} = p_a \delta^{\alpha\beta}$ , where  $a$  stands for species of particles. Equations (1) and (3) are continuity equations for electrons and ions correspondingly. These equations show conservation of particle number of electrons and ions. Equations (2) and (4) are the momentum balance (Euler) equations for electrons and ions. The first terms in the left-hand side of Euler equations  $m_a n_a (\partial_t + \mathbf{v}_a \nabla) \mathbf{v}_a$  are the kinematic part. The second terms are the gradient of the thermal pressure or the Fermi pressure for degenerate electrons and ions. It appears as the thermal part of the momentum flux related to distribution of particles on states with different momentum. The third terms are the quantum Bohm potential appearing as the quantum part of the momentum flux. In the right-hand sides of the Euler equations we present interparticle interaction and interaction of particles with external electromagnetic fields. The first group of terms in the right-hand side of the Euler equations describe interaction with the external electromagnetic fields. The second term in the Euler equation for electrons (2) describes the electron-electron Coulomb interaction. The third term is the Coulomb action of ions on electron motion. The second term in the Euler equation

for ions (4) gives the ion-ion Coulomb interaction. The Coulomb field of electrons acting on ions is presented by the third term in equation (4). The last terms in the right-hand sides of equations (2) and (4) describes the Coulomb electron-electron and ion-ion exchange interactions  $\mathbf{F}_{C,a}$  correspondingly. These two terms are main subject of this paper. Below we consider their contribution in the plasma wave dispersion.

We can rewrite the Euler equations (2) and (4) in terms of the self-consistent electric field

$$\mathbf{E}_{a(int)} = -q_a \nabla \int G(\mathbf{r}, \mathbf{r}') n_a(\mathbf{r}', t) d\mathbf{r}', \quad (5)$$

where  $\mathbf{E}_{a(int)}$  is the electric field created by particles of species  $a = e, i$ . Hence equations (2), (4) attain more familiar form

$$\begin{aligned} m_e n_e (\partial_t + \mathbf{v}_e \nabla) \mathbf{v}_e + \nabla p_e - \frac{\hbar^2}{2m_e} n_e \nabla \left( \frac{\Delta n_e}{n_e} - \frac{(\nabla n_e)^2}{2n_e^2} \right) \\ = q_e n_e \left( \mathbf{E}_{ext} + \mathbf{E}_{int} + \frac{1}{c} [\mathbf{v}_e, \mathbf{B}_{ext}] \right) + \mathbf{F}_{C,e}, \end{aligned} \quad (6)$$

and the Euler equation for ions

$$\begin{aligned} m_i n_i (\partial_t + \mathbf{v}_i \nabla) \mathbf{v}_i + \nabla p_i - \frac{\hbar^2}{2m_i} n_i \nabla \left( \frac{\Delta n_i}{n_i} - \frac{(\nabla n_i)^2}{2n_i^2} \right) \\ = q_i n_i \left( \mathbf{E}_{ext} + \mathbf{E}_{int} + \frac{1}{c} [\mathbf{v}_i, \mathbf{B}_{ext}] \right) + \mathbf{F}_{C,i}. \end{aligned} \quad (7)$$

Physical meaning of terms in the non-integral Euler equations (6) and (7) is similar to described above for equations (2) and (4). Interparticle interactions are presented in (6) and (7) in terms of internal electric field satisfying the Maxwell equations.

The Maxwell equations  $\nabla \times \mathbf{E}_{int} = 0$  and  $\nabla \mathbf{E}_{int} = 4\pi \rho$ , where  $\rho_{3D} = \sum_a q_a n_a$ , and  $\rho_{2D} = \delta(z) \sum_a q_a n_{a,(2D)}$ , with  $\mathbf{E}_{int} = \sum_a \mathbf{E}_{a(int)}$ . Presented here two dimensional charge density  $\rho_{2D} = \rho_{2D}(x, y, z)$  is explicitly presented as two dimensional layer in the three dimensional physical space. Two dimensional particle concentration is a function of two space coordinates  $n_{a,(2D)} = n_{a,(2D)}(x, y)$  in plane  $z = 0$ .

Equation of state for degenerate 2D Fermi gas is  $p_{a,2D} = \pi \hbar^2 n_{a,2D}^2 / (2m_a)$ . In 3D case one similarly finds  $p_{a,3D} = (3\pi^2)^{2/3} \hbar^2 n_{a,3D}^{5/3} / (5m_a)$ . These equations of state take place for unpolarized fermions at zero temperature  $p_{a,ND} = p_{a,ND\uparrow\downarrow}$ ,  $N = 2$  or  $3$ , where subindex  $\uparrow\downarrow$  means that in each occupied quantum state we have two particles with opposite spins. Hence we have two fermions in each state with energy lower than the Fermi energy  $\varepsilon_F$ , fermions of each pair have opposite spins. When system of spin-1/2 fermions is polarized then distribution of fermions looks like one electron in each state with energy lower than  $2^{2/3} \varepsilon_{F,3D}$  for 3D mediums, and

$2\varepsilon_{F,2D}$  for 2D mediums. For polarized systems equations of state appears as  $p_{a,3D\uparrow\uparrow} = 2^{2/3} (3\pi^2)^{2/3} \hbar^2 n_{a,3D}^{5/3} / (5m_a)$  for 3D mediums, and  $p_{a,2D\uparrow\uparrow} = 2\pi \hbar^2 n_{a,2D}^2 / (2m_a)$  for 2D mediums, where subindex  $\uparrow\uparrow$  means that all particles have same spin direction. We may consider partly polarized particles, then we need to introduce ratio of polarizability  $\eta = \frac{|n_\uparrow - n_\downarrow|}{n_\uparrow + n_\downarrow}$ , with indexes  $\uparrow$  and  $\downarrow$  means particles with spin up and spin down. Here we have that instead  $p_0 = p_\uparrow + p_\downarrow$ , with  $p_\uparrow = p_\downarrow = p_0/2$  for  $\eta = 0$ , we find  $p = \tilde{p}_\uparrow + \tilde{p}_\downarrow$ , with  $\tilde{p}_\uparrow = 2^{5/3} p_\uparrow = 2^{2/3} p_0$  and  $\tilde{p}_\downarrow = 0$  for  $\eta = 1$  in 3D case. Similarly we have  $p_0 = p_\uparrow + p_\downarrow$ , with  $p_\uparrow = p_\downarrow = p_0/2$  at  $\eta = 0$ , we obtain for  $\eta = 1$   $p = \tilde{p}_\uparrow + \tilde{p}_\downarrow$ , with  $\tilde{p}_\uparrow = 4p_\uparrow = 2p_0$  and  $\tilde{p}_\downarrow = 0$  for 2D case. In general case of partially polarized system of particles we can write  $p_{a,3D\uparrow\uparrow} = \vartheta_{3D} (3\pi^2)^{2/3} \hbar^2 n_{a,3D}^{5/3} / (5m_a)$  for 3D mediums, and  $p_{a,2D\uparrow\uparrow} = \vartheta_{2D} \pi \hbar^2 n_{a,2D}^2 / (2m_a)$  for 2D mediums, with

$$\vartheta_{3D} = \frac{1}{2} [(1 + \eta)^{5/3} + (1 - \eta)^{5/3}], \quad (8)$$

and

$$\vartheta_{2D} = 1 + \eta^2, \quad (9)$$

where  $\uparrow\uparrow$  stands for partially polarized systems, that means that part of states contain two particle with opposite spins and other occupied states contain one particle with same spin direction.

Considering two electrons one finds that full wave function is anti-symmetric. If one has two electrons with parallel spins one has that wave function is symmetric on spin variables, so it should be anti-symmetric on space variables. In opposite case of anti-parallel spins one has anti-symmetry of wave function on spin variables and symmetry of wave function on space variables.

Considering energy of two electron Coulomb interaction one finds it two parts: the classic like part C and the exchange part A. For the parallel (anti-parallel) spins one obtains  $E_{\uparrow\uparrow} = C - A$  ( $E_{\uparrow\downarrow} = C + A$ ). Parallel (anti-parallel) configuration of spins decreases (increases) energy of the Coulomb interaction.

Systems of unpolarized electrons then average numbers of electrons with different direction of spins equal to each other, we find that average number of particles for a chosen with parallel and anti-parallel spins is the same. Consequently we have that average exchange interaction equals to zero.

In partly polarized systems the numbers of particles with different spin are not the same. In this case a contribution of the average exchange interaction appears. At full measure it reveals in fully polarized system then all electrons have same direction of spins. In accordance with the previous discussion we find that exchange interaction, for this configuration, gives attractive contribution in the force field.

This result allows us to find the force field of the

Coulomb exchange interaction

$$\begin{aligned}\mathbf{F}_{C,a(3D)} &= 2^{4/3} q_a^2 \sqrt[3]{\frac{3}{\pi}} \sqrt[3]{n_a} \nabla n_a \\ &= 6 q_a^2 \sqrt[3]{\frac{6}{\pi}} n_a \nabla n_a^{1/3}.\end{aligned}\quad (10)$$

We obtain the force field of exchange interaction for 2D quantum plasmas in the following form

$$\begin{aligned}\mathbf{F}_{C,a(2D)} &= 2^{3/2} \sqrt{2\pi} \frac{24 \text{arsh} 1}{\pi^2} q_a^2 \sqrt{n_a} \nabla n_a \\ &= 8 \sqrt{\pi} \frac{\beta}{\pi^2} q_a^2 n_a \nabla \sqrt{n_a}.\end{aligned}\quad (11)$$

where we introduce  $\beta \equiv 24 \text{arsh} 1 = 21.153$ .

The force fields (10) and (11) are obtained for fully polarized systems of identical particles.

For partially polarized particles the force fields reappear as  $\mathbf{F}_{C,a(3D)} = \zeta_{3D} q_a^2 \sqrt[3]{\frac{3}{\pi}} \sqrt[3]{n_a} \nabla n_a$  and  $\mathbf{F}_{C,a(2D)} = \zeta_{2D} \sqrt{2\pi} \frac{\beta}{\pi^2} q_a^2 \sqrt{n_a} \nabla n_a$ , with

$$\zeta_{3D} = (1 + \eta)^{4/3} - (1 - \eta)^{4/3} \quad (12)$$

and

$$\zeta_{2D} = (1 + \eta)^{3/2} - (1 - \eta)^{3/2}. \quad (13)$$

We should mention that coefficients  $\zeta_{3D} \sim \eta$  and  $\zeta_{2D} \sim \eta$  are proportional to spin polarization. Limit cases of  $\zeta_{3D}$  and  $\zeta_{2D}$  are  $\zeta_{3D}(0) = 0$ ,  $\zeta_{3D}(1) = 2^{4/3}$ ,  $\zeta_{2D}(0) = 0$  and  $\zeta_{2D}(1) = 2^{3/2}$ .

If we do not apply a magnetic field to systems under consideration we need to have systems, where self-organization of equilibrium state leads to formation of population levels with uncompensated spin, such it is in ferromagnetic domains. Otherwise we have no contribution of the Coulomb exchange interaction.

Force fields of exchange interaction (10) and (11) are potential fields. Thus they do not give contribution in dispersion of transverse waves. They affect longitudinal waves and waves with complex polarization: longitudinal-transverse waves. Consequently electromagnetic waves are not affected by the Coulomb exchange interaction in 3D and 2D mediums in absence of external fields.

Force fields of the Coulomb exchange interaction can be presented as product of the particle concentration on the gradient of a function when it gives no contribution in equations of the vorticity evolution (see Refs. [29], [48], [49], and [50]).

Many-particle quantum hydrodynamic equations can be represented in form of the non-linear Schrodinger equation (NLSE). Let us consider evolution of electrons

at motionless ions. We introduce the wave function in medium defined in terms of hydrodynamic variables

$$\Phi = \sqrt{n} \exp\left(i \frac{mS}{\hbar}\right), \quad (14)$$

where  $S$  is the potential of velocity field. Let us mention that the NLSE can be derived for eddy-free motion of electrons. Definition (14) can be applied for three dimensional and low dimensional systems of particles  $|\Phi_{3D}| = \sqrt{n_{3D}}$ , and  $|\Phi_{2D}| = \sqrt{n_{2D}}$ , with  $[n_{3D}] = \text{cm}^{-3}$ ,  $[n_{2D}] = \text{cm}^{-2}$ . To derive NLSE we need to differentiate function (14) with respect to time. After some calculations we find NLSE for electrons in three dimensional quantum plasmas

$$\begin{aligned}i\hbar\partial_t\Phi_{3D} &= \left(-\frac{\hbar^2\Delta}{2m_e} + \vartheta_{3D} \frac{(3\pi^2)^{2/3}\hbar^2 n_e^{2/3}}{2m_e} \right. \\ &\quad \left. - 3\sqrt[3]{\frac{3}{\pi}} \zeta_{3D} q^2 n^{1/3} + q_e \varphi\right) \Phi_{3D}.\end{aligned}\quad (15)$$

For 2DEGs we obtain

$$\begin{aligned}i\hbar\partial_t\Phi_{2D} &= \left(-\frac{\hbar^2\Delta}{2m_e} + \vartheta_{2D} \frac{\pi\hbar^2 n}{m_e} \right. \\ &\quad \left. - \frac{2\beta\sqrt{2\pi}}{\pi^2} \zeta_{2D} q_e^2 \sqrt{n} + q_e \varphi\right) \Phi_{2D}.\end{aligned}\quad (16)$$

Equations (15) and (16) contain potential of the electric field  $\varphi$  presenting the external and internal electric fields:  $\mathbf{E} = -\nabla\varphi$ .

Considering dynamic of two or more species we should present the wave functions in medium for each species and derive NLSEs for each species either.

### III. APPLICATIONS

In this section we consider small perturbations of equilibrium state describing by nonzero particle concentration  $n_0$ , and zero velocity field  $\mathbf{v}_0 = 0$  and electric field  $\mathbf{E}_0 = 0$ .

Assuming that perturbations are monochromatic

$$\begin{pmatrix} \delta n \\ \delta \mathbf{v} \\ \delta \mathbf{E} \end{pmatrix} = \begin{pmatrix} N_A \\ \mathbf{V}_A \\ \mathbf{E}_A \end{pmatrix} e^{-i\omega t + i\mathbf{k}\mathbf{r}}, \quad (17)$$

we get a set of linear algebraic equations relatively to  $N_A$  and  $\mathbf{V}_A$ . Condition of existence of nonzero solutions for amplitudes of perturbations gives us a dispersion equation.



### A. Three dimensional quantum plasmas

In classic plasmas the Langmuir waves have the following spectrum

$$\omega^2 = \omega_{Le}^2 + \frac{\gamma T}{m_e} k^2, \quad (18)$$

with the three dimensional Langmuir frequency

$$\omega_{Le,3D}^2 = \frac{4\pi e^2 n_{0,3D}}{m_e}, \quad (19)$$

$T$  is the temperature,  $\gamma$  is the adiabatic index.

We consider quantum plasmas, so we are interested in the low temperature properties, then carriers are degenerated. Hence we have contribution of the Fermi pressure instead of the temperature.

Spectrum of the Langmuir waves in 3D quantum plasmas with the Coulomb exchange interaction appears as

$$\begin{aligned} \omega^2 = & \omega_{Le,3D}^2 - \zeta_{3D} \sqrt[3]{\frac{3}{\pi}} \frac{e^2}{m_e} \sqrt[3]{n_{0e}} k^2 \\ & + \vartheta_{3D} \frac{(3\pi^2)^{2/3} \hbar^2 n_{0e}^{2/3}}{3m_e^2} k^2 + \frac{\hbar^2 k^4}{4m_e^2}. \end{aligned} \quad (20)$$

The first term in formula (20) is the 3D Langmuir frequency existing due to the Coulomb interaction in the self-consistent field approximation. The second term describes the Coulomb exchange interaction between electrons. The third term appears from the Fermi pressure. The last term describes contribution of the quantum Bohm potential.

Let us consider a dimensionless form of the spectrum of 3D Langmuir waves. Introducing dimensionless frequency  $\Omega = \omega/\omega_{Le,3D}$  and wave vector  $\xi = k/\sqrt[3]{n_{0e,3D}}$  we obtain

$$\begin{aligned} \Omega^2 = & 1 - \zeta_{3D} \frac{1}{4\pi} \sqrt[3]{\frac{3}{\pi}} \xi^2 \\ & + \vartheta_{3D} \left( \frac{(3\pi^2)^{2/3}}{3} + \frac{1}{16\pi} \xi^2 \right) \Lambda_{3D} \xi^2, \end{aligned} \quad (21)$$

where we also have a dimensionless parameter

$$\Lambda_{3D} = \frac{\hbar^2}{m_e e^2} \sqrt[3]{n_{0e}}, \quad (22)$$

depending on fundamental constants  $\hbar$ ,  $e$ ,  $m_e$ , and the equilibrium concentration of electrons  $\sqrt[3]{n_{0e,3D}}$ . The contribution of the exchange interaction can be considered as a shift of the Fermi pressure since both of them are proportional to  $\xi^2$ . On the other hand, the term describing the exchange interaction is proportional to the square of the Langmuir frequency. From this point of view we see that exchange interaction gives a shift of the Langmuir

frequency and this shift is proportional to square of the wave vector  $\xi^2$ . So exchange interaction is considerable in the short wave length limit.

Increasing of the particle concentration allows to increase maximal wave vector of wave propagation in the medium. Hence, in short wave length limit the first two terms reveal same dependence on the equilibrium particle concentration. For whole range of wave vectors we see that the first term  $\sim n_{0e,3D}$  grows faster than the second term  $\sim n_{0e,3D}^{1/3}$  with the increasing of the particle concentration. The third term has an intermediate rate of grow being proportional to  $n_{0e,3D}^{2/3}$ . However, in the short wave length limit the third and fourth terms increase rather faster like  $n_{0e,3D}^{2/3} k^2$  and  $k^4$  correspondingly. Finally formula (21) shows that the third and fourth terms can be rather large at high densities (particle concentrations) and large wave vectors.

Let us estimate contribution of the exchange interaction in compare with other terms in dispersion dependence of the Langmuir waves. Comparing exchange interaction with the Fermi pressure we should consider ratio of the second term to the third term

$$\chi_{EF} = \frac{m_e e^2}{\hbar^2 \sqrt[3]{n_{0e,3D}}} = \frac{1}{\Lambda_{3D}} \approx \sqrt[3]{\frac{10^{25}}{n_{0e,3D}}}. \quad (23)$$

Exchange interaction prevails over the Fermi pressure than  $\chi_{EF} > 1$ , that corresponds to  $n_{0e,3D} < 10^{25} \text{cm}^{-3}$ . Hence the Coulomb exchange interaction is larger than the Fermi pressure in metals  $n_{0e,3D} \sim 10^{22} \text{cm}^{-3}$  and semiconductors  $n_{0e,3D} \sim 10^{18} \text{cm}^{-3}$ . Considering extreme astrophysical objects like white dwarfs  $n_{0e,3D} \sim 10^{28} \text{cm}^{-3}$  we find that Fermi pressure is larger than the Coulomb exchange interaction. Ratio between the Fermi pressure and the exchange interaction does not depend on the wave vector  $k$ .

We have considered high frequency waves. Next step is consideration of the low frequency excitations

$$\begin{aligned} \omega_{3D}(k) = & kv_{s,3D} \sqrt{1 - \frac{\zeta_{3D}}{\vartheta_{3D}} \frac{3}{4\pi} \sqrt[3]{\frac{3}{\pi}} \frac{\omega_{Le,3D}^2}{n_{0e,3D}^{2/3} v_{Fe,3D}^2}} \times \\ & \times \frac{1}{\sqrt{1 + (kr_{De,3D})^2 \left( 1 - \frac{\zeta_{3D}}{\vartheta_{3D}} \frac{3}{4\pi} \sqrt[3]{\frac{3}{\pi}} \frac{\omega_{Le,3D}^2}{n_{0e,3D}^{2/3} v_{Fe,3D}^2} \right)}}, \end{aligned} \quad (24)$$

where  $v_{s,3D} = \sqrt{m_e/m_i} \sqrt{\vartheta_{3D}} \cdot v_{Fe,3D}/3$  is the three dimensional velocity of sound,  $r_{De,3D} = \sqrt{\vartheta_{3D} v_{Fe,3D}} / (3\omega_{Le,3D})$  is the Debye radius. In formula (24) and similar formulas below we extract contribution of the Fermi pressure. Hence formulas for ion-acoustic waves contains well-known contribution of the pressure multiplied by factor showing contribution of exchange interaction.

In the long wavelength limit we have

$$\omega(k) = kv_{s,3D} \sqrt{1 - \frac{\zeta_{3D}}{\vartheta_{3D}} \frac{3}{4\pi} \sqrt{\frac{3}{\pi}} \frac{\omega_{Le,3D}^2}{n_{0e,3D}^{2/3} v_{Fe,3D}^2}}. \quad (25)$$

In the short wavelength limit we find

$$\omega^2(k) = \omega_{Li,3D}^2. \quad (26)$$

From formula (26) we see that the exchange interaction gives no contribution in the ion-acoustic waves in the short wave length limit.

### B. Two dimensional quantum plasmas: two dimensional electron gas and ion motion contribution

Two dimensional quantum plasmas are systems of electrons and ions being under confinement, so we have plane-like objects. 2D quantum plasmas are surrounded by medium. However, main properties can be obtained considering a 2D layer in empty 3D space. Such objects as 2DEG (two dimensional electron gas) and 2DHG (two dimensional hole gas) are common objects in physics of semiconductors. As application these objects appears as parts of transistors. Consideration of 2DEG and 2DHG corresponds to description of high frequency excitations. In this section we also include ion dynamics.

Spectrum of high frequency longitudinal excitations, which are the Langmuir waves, appears as follows

$$\begin{aligned} \omega^2 = & \omega_{Le,2D}^2 - \zeta_{2D} \frac{\beta \sqrt{2\pi} e^2}{\pi^2 m_e} \sqrt{n_{0e}} k^2 \\ & + \vartheta_{2D} \frac{\pi \hbar^2 n_{0e}}{m_e^2} k^2 + \frac{\hbar^2 k^4}{4m_e^2}, \end{aligned} \quad (27)$$

where we have used the two dimensional Langmuir frequency

$$\omega_{Le,2D}^2 = \frac{2\pi e^2 k n_{0,2D}}{m_e} \sim k, \quad (28)$$

which is not a constant, but it is proportional to the wave vector  $k$ .

Similarly to the three dimensional spectrum (see formula (20)) different terms in formula (27) have the following meaning: self-consistent Coulomb interaction, the exchange Coulomb interaction, the Fermi pressure, and the quantum Bohm potential.

Next we discuss some properties of the 2D Langmuir wave spectrum.

Let us consider the exchange interaction with the Fermi pressure for 2D quantum plasmas. To this end we introduce the following dimensionless parameter

$$\chi_{EF,2D} = \frac{m_e e^2}{\hbar^2 \sqrt{n_{0e,2D}}} \approx \sqrt{\frac{10^{16}}{n_{0e,2D}}}. \quad (29)$$

In 2D semiconductor objects  $n_{0e,2D} \ll 10^{16} \text{cm}^{-2}$ . Consequently, the exchange interaction plays significant role in collective properties of semiconductors.

To this end we present a dimensionless form of formula (27) introducing dimensionless parameters  $\Omega_{2D} = \omega \sqrt{m/(2\pi e^2 n_{0e,2D}^{3/2})}$  and  $\xi_{2D} = k/\sqrt{n_{0e,2D}} \sim ak$ , with  $a$  is the average interparticle distance. Hence we have

$$\begin{aligned} \Omega_{2D}^2 = & \xi \left( 1 - \zeta_{2D} \frac{\beta}{\sqrt{2\pi} \pi^2} \xi \right) \\ & + \frac{1}{2} \vartheta_{2D} \Lambda_{2D} \xi^2 \left( 1 + \frac{1}{4\pi} \xi^2 \right), \end{aligned} \quad (30)$$

with

$$\Lambda_{2D} = \frac{\hbar^2}{m e^2} \sqrt{n_{0e,2D}}. \quad (31)$$

Spectrum of the 2D ion-acoustic waves in presence of the exchange interaction appears as

$$\begin{aligned} \omega_{2D}(k) = & kv_{s,2D} \sqrt{1 - \frac{\zeta_{2D}}{\vartheta_{2D}} \frac{2\beta \sqrt{2\pi} e^2 \sqrt{n_{0e,2D}}}{\pi^2 m v_{Fe,2D}^2}} \times \\ & \times \frac{1}{\sqrt{1 + (kr_{De,2D}(k))^2 \left( 1 - \frac{\zeta_{2D}}{\vartheta_{2D}} \frac{2\beta \sqrt{2\pi} e^2 \sqrt{n_{0e,2D}}}{\pi^2 m v_{Fe,2D}^2} \right)}}, \end{aligned} \quad (32)$$

with  $v_{s,2D} = \sqrt{m_e/m_i} \sqrt{\vartheta_{2D}} \cdot v_{Fe,2D}/2$  is the two dimensional velocity of sound,  $r_{De,2D} = \sqrt{\vartheta_{2D} v_{Fe,2D}} / (2\omega_{Le,2D})$ . The spectrum of the 2D ion-acoustic waves can be written in more explicit form, which shows dependence on the wave vector  $k$

$$\begin{aligned} \omega_{2D}(k) = & kv_{s,2D} \sqrt{1 - \frac{\zeta_{2D}}{\vartheta_{2D}} \frac{2\beta \sqrt{2\pi} e^2 \sqrt{n_{0e,2D}}}{\pi^2 m v_{Fe,2D}^2}} \times \\ & \times \frac{1}{\sqrt{1 + kD \left( 1 - \frac{\zeta_{2D}}{\vartheta_{2D}} \frac{2\beta \sqrt{2\pi} e^2 \sqrt{n_{0e,2D}}}{\pi^2 m v_{Fe,2D}^2} \right)}}, \end{aligned} \quad (33)$$

where

$$D = \frac{\hbar}{2\sqrt{2\pi} e^2 \sqrt{n_{0e,2D}}}. \quad (34)$$

In the long wavelength limit we have

$$\omega(k) = kv_{s,2D} \sqrt{1 - \frac{\zeta_{2D}}{\vartheta_{2D}} \frac{2\beta \sqrt{2\pi} e^2 \sqrt{n_{0e,2D}}}{\pi^2 m v_{Fe,2D}^2}}. \quad (35)$$

In the short wavelength limit we find

$$\omega^2(k) = \omega_{Li,2D}^2 \sim k. \quad (36)$$

As in 3D case we find no contribution of the exchange interaction in the short wave length ion-acoustic waves.

#### IV. CONCLUSIONS

We have briefly described quantum hydrodynamic model for electron-ion quantum plasmas with exchange interaction. We have considered three and two dimensional electron-ion quantum plasmas. We have derived explicit form of the Coulomb exchange interaction force field for electron-electron and ion-ion Coulomb interaction. We have applied this model to dispersion properties of electron-ion plasmas tracing contribution of the exchange interaction. We have considered small ampli-

tude linear excitations in absence of external field. However obtained results open possibilities for consideration of nonlinear effects. Derived force fields for the Coulomb exchange interaction can be used to study various effects in magnetized plasmas as well.

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